Vagueness via Nonclassical Logics

17–20 December 2014
University of Sydney

Timetable and Abstracts

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Timetable

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Abstracts

Libor Běhounek, Petr Cintula and Carles Noguera (I)

Fuzzy logics as logics of linearly ordered resources I: Motivation and apparatus

Linear logic [3] was proposed as a resource-sensitive logical system. The main idea was that, thanks to the absence of the contraction rule, each time a premise is used in an inference, it is *spent*. Premises can thus be understood as resources needed to support the conclusion, as illustrated by Girard's Marlboro–Camel example: if I have one dollar, I can buy a pack of Marlboros ($D \to M$), and if I have one dollar, I can buy a pack of Camels ($D \to C$), but from this we cannot infer that if I have one dollar, I can buy a pack of Malboros and a pack of Camels ($D \to M \mathbin{\&} C$); we would need to use the premise twice, i.e. we can only infer that $D \mathbin{\&} D \to M \mathbin{\&} C$. The lack of contraction yields the non-idempotent behavior of the conjunction &.

Since this resource-sensitiveness of linear logic is due solely to the absence of the contraction rule, in principle, any contraction-free substructural logic could be interpretable in terms of resources as well. The choice of one or another contraction-free logic may depend on the nature of the resources one wants to deal with. Prototypical examples are money (costs, prices), material goods (e.g., construction materials, cooking ingredients), computer resources (e.g., disk space or computation time), as well as sets, multisets, or tuples thereof.

It can be observed that the sizes of prototypical kinds of resources can be linearly ordered (money), or at least decomposed into linearly ordered components (cooking ingredients). Following [1] we shall argue that such resources form algebraic structures known as (semi)linear residuated lattices, whose operations represent comparison and composition of resources. Since classes of (semi)linear residuated lattices constitute the algebraic semantics of fuzzy logics [2, 4], prototypical kinds of resources are actually governed by suitable fuzzy logics rather than linear logic; for instance, linearly ordered or linearly decomposable resources validate the se-called prelinearity law, $(A \to B) \vee (B \to A)$, which is not valid in linear logic.

The purpose of the talk is to elaborate the motivation sketched above and substantiate in details the basic technical notions that allow fuzzy logics to be seen as logics of (linearly ordered) resources, taking into account the present state of development of Mathematical Fuzzy Logic.

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Libor Běhounek, Petr Cintula and Carles Noguera (II)

Fuzzy logics as logics of linearly ordered resources II: Applications

The rendering of fuzzy logics as logics of linearly ordered resources (see Part I of this abstract) opens the way for applications of fuzzy logic in resource-aware reasoning. In particular, the notion of *feasibility* of a task can adequately be handled by fuzzy logic if the degree of feasibility is understood as proportional to the resources needed for completing the task. In this talk we showcase the usefulness of handling the cost of tasks by means of fuzzy logic in: (i) elimination of various instances of the sorites paradox to which feasibility-related notions are susceptible, such as the logical omniscience paradox for feasible knowledge in standard epistemic logic, and (ii) modeling the cost of program runs in propositional dynamic logic.

The logical omniscience paradox consists in the fact that the closure of a rational agent's knowledge under logical rules of inference makes all logical consequences of the agent's actual knowledge part of the agent's knowledge—which is quite unrealistic for real-world agents. For a logical analysis of the paradox it is important to distinguish three types of knowledge: the *actual* knowledge, which is immediately available to the agent (e.g., the contents of the agent's database); the *potential* knowledge, which is (in principle) derivable from the agent's actual knowledge by logical inference; and the *feasible* knowledge, which the agent is effectively able to infer from the actual knowledge. Logical omniscience is only counter-intuitive for feasible knowledge, as the potential knowledge indeed contains all logical consequences of the agent's actual knowledge, and the actual knowledge is not closed under logical rules of inference.

Arguably, the logical omniscience paradox for feasible knowledge is actually an instance of the sorites paradox (if the agent can feasibly perform n steps of inference, then for sure they can perform $n+1$ steps as well). Elaborating [2] we argue that our cost-based interpretation of fuzzy logics is especially suitable for handling this instance of sorites paradox: we propose to interpret the degree of feasibility of a knowledge as proportional to the resources (e.g., time, memory, etc.) the agent needs to spend in order to derive this knowledge from the agent's actual knowledge. The ensuing gradual notion of feasible knowledge models the fact that some pieces of knowledge can be derived more easily from the actual knowledge than others. The apparatus of fuzzy logic then handles the logical omniscience paradox in the same manner it handles the sorites paradox [3].

Our second showcase application regards dynamic logics, i.e., a family of modal logics designed for reasoning about computer programs [4]. Our starting point is the natural assumption that each run of a program (which in classical dynamic logic can only get the computation from one state to another) has also some *cost* (with linearly ordered components, e.g., in terms of computational time, necessary memory resources, etc.). Based on this idea we present a suitable notion of fuzzy dynamic logic (originally introduced in [1]). In particular, by taking fuzzy accessibility relations R_α expressing the costs of performing the program α, the formulae $[\alpha]\varphi$ and $\langle\alpha\rangle\varphi$ are given a natural cost-sensitive meaning when evaluated by the semantical rules of fuzzy logic, with a correspondence between the connectives of fuzzy logic and compositions of programs. The apparatus of fuzzy dynamic logic is then capable of, e.g., expressing the costs of programs (as the truth values of the formulae $[\alpha]\top$), verifying the correctness of feasible runs of a program (by almost disregarding the states only achievable by too costly runs), and providing an apparatus for speaking about vague conditions, for example the correctness of a program for *small* inputs.

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Marta Bílková

A coalgebraic approach to many-valued modal logics

Abstract theory of coalgebras has recently became one of most important bridges between modal logic and computer science: from a logician's point of view it provides a new level of generality for studying various modal logics, while from a computer-scientist's point of view it provides a general framework for designing expressive (modal) languages describing behavior of abstract transition systems modeled as coalgebras. It is natural to ask what benefits a coalgebraic approach to many-valued modal logics may have: from a logician's point of view, it allows to generalize logics of many-valued Kripke-style relational semantics to the coalgebraic level, offering in particular an answer to the question what is the least modal logic over a given residuated lattice $\mathcal{V}$, in which valuations and the accessibility relation take values [4]. From a computer-scientist's point of view it allows to generalize logics of abstract transition systems modeled as coalgebras (of which Kripke frames are a special case) to the many-valued setting, allowing for a many-valued observable phenomena to be captured in the framework. We outline two approaches of designing an expressive logical language for coalgebras common in coalgebraic logic, and apply them in a many-valued setting — the first is based on an idea of L. Moss and deals with a nonstandard syntax, the second is based on a connection between syntax (modal algebras) and semantics (coalgebras) called logical connection, in the boolean case based on the Stone duality.

Moss' coalgebraic logic, introduced in Moss' pioneering paper [6], extends the underlying propositional boolean logic with a single modality whose arity is given by the coalgebra functor (the type of transitions) and whose semantics is given by a lifting of the local satisfaction relation with the coalgebra functor. Despite its nonstandard syntax, the language is easy to deal with. In particular, it allows for a simple proof of the Hennessy-Milner property — it is expressive for bisimilarity. It is natural to investigate possibilities of extending the results beyond the boolean setting. We first consider set-based coalgebras with many-valued valuations (of which Kripke frames with a many-valued accessibility relation are a special case) and show that the resulting finitary Moss' logic has the Hennessy-Milner property — it is adequate and expressive for a crisp notion of bisimilarity based on many-valued bisimulations defined via the many-valued relation lifting [1]. Next we discuss coalgebras in a different category $\mathcal{V}$-cat of generalized metric spaces [2] allowing for a many-valued notion of bisimilarity, and address similar issues there. In both cases it seems that well-behaved relation lifting allowing for the results to go through is available for a large class of coalgebra functors provided the residuated lattice is a complete Heyting algebra.

The approach based on a *logical connection* [3, 5] between modal algebras and coalgebras has a benefit of providing one with an abstract machinery producing an expressive language based on standard modalities, and allowing to derive from the general picture algebraic completeness for the resulting modal logic. We will show particular examples of logical connections and discuss the resulting modal logics. This part is still work in progress.

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Mark Colyvan

The Sorites is a Liar

There has been a great deal of recent interest in the similarities between the liar paradox and the sorites paradox. This work has arisen from a number of different directions. First there are those who argue that a kind of indeterminacy lies at the heart of both. There are those interested in paraconsistent approaches to the sorites and forcing a connection with the liar strengthens the case for a uniform (paraconsistent) treatment of the two. Finally, work on generalisations of the sorites to more abstract metric spaces and even topological spaces reveals hitherto unnoticed structural similarities. In this paper I will put forward a case for the liar and the sorites being of a kind. I will draw on the work coming from these three different projects and show that they all lead to the same surprising conclusion: the liar and the sorites are very much alike.

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Chris Fermüller

Vagueness and logical dialogue games

Already in the 1980s Robin Giles has proposed to use a particular logical dialogue game, originally design to model formal reasoning in physics, to provide a semantics for a logic of vague predicates and propositions. This logic turns out to coincide with infinite-valued Lukasiewicz logic. Giles' approach consists of two parts: (1) the assignment of intermediate truth values to atomic formulas via success/failure probabilities of associated 'dispersive experiments' and (2) the extraction of truth functions for logical connectives from rules for the systematic reduction of arguments about logical complex statements to arguments about atomic statements. We will re-visit Giles' game from a more general perspective, reviewing attempts to characterize other fuzzy logics than Lukasiewicz logic in a similar fashion. More importantly, we discuss ideas to connect the concept of 'dispersive experiments' with various theories of vagueness, in particular supervaluationism, (fuzzy and non-fuzzy) plurivaluationsm and contextualism. We will emphasize two lessons learned from this enterprise: (1) Dialogue games provide a basis for justifying the use of a few particular, but by no means all, truth-functional many-valued logics in the context of models for reasoning involving vague concepts. (2) The emerging fuzzy logics do not, by themselves, already constitute a particular theory of vagueness. They are rather to be understood as formal structures that may arise in more than one way from different theories of vague language.

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Tomasz Kowalski

Relation Algebras, Qualitative Calculi and syllogistics of approximate propositions

Classical syllogistics can be interpreted in a natural way as an algebra of certain binary relations. This much was known to De Morgan, Schroeder and Peirce. It is easily seen that this algebra of relations is in fact isomorphic to the algebra of relations underlying RCC5: a qualitative calculus used in spatial reasoning. Similar qualitative calculi are quite widely employed in some areas of AI.

This suggests that extensions of syllogistics could be modelled in a simiar way. I will present one such extension, into which approximate propositions like "many S are P" or "only a few S are not P" can be incorporated. I will focus on mathematically interesting aspects: representations and computational complexity.

Antonio Ledda and Giuseppe Sergioli

Vagueness, uncertainty and fuzzyness in quantum computational logics

The usual notion of *uncertainty* seems to be tightly related to an epistemic condition [12]. A typical case: a coin was flipped, but a specified knower does not see which side of the coin faced up when it landed. It seems to be generally accepted that uncertainty deals with ignorance: a certain predicate is uncertain, with respect to a specific knower, if the information available is not sufficient to determine its applicability.

On the other hand, *vagueness* seems to have little to do with ignorance. Instead, this notion often refers to concepts whose extensions are lacking in clarity [11, 7, 13]. Natural examples of vague concepts are predicates admitting "border-line cases", in which it is hard to sharply determine whether an object falls completely in the extension of the predicate or not. Rather, an object may possess specific properties to some extent. A successful framework for dealing with vagueness is provided by the so-called *many-valued/fuzzy logics* [9, 10].

In the microscopic domain, appreciable overlaps between the concepts of uncertainty and vagueness are to mention. A remarkable example is the "double-slit experiment". A coherent light source is placed in front of a screen pierced by two parallel slits of the same size. By the wave nature of light, the light waves passing through the slits interfere, producing bright and dark bands on the screen behind the slits. In quantum mechanics, this interference phenomenon is expressed by a superposition state $|\psi\rangle = a|0\rangle + b|1\rangle$, where $|\psi\rangle$ represents the state of a photon with probabilities $|a|^2$, $|b|^2$ to be measured either behind the first slit $|0\rangle$, or behind the second slit $|1\rangle$, respectively. The state $|\psi\rangle$ is a *pure state*, that represents a maximal piece of information that cannot be increased by any further observation. However, once the observable is fixed, by its very nature, $|\psi\rangle$ involves an amount of uncertainty, which in this case is a property of the state not related to observer's knowledge. A state may also represent non-maximal pieces of information, that is: an amount of information that can be improved by further observations. In this case the state is said to be *mixed*. A generic state is mathematically represented by a density operator.

We will see that, at the microscopic level, uncertainty and vagueness can be captured under several degrees of freedom rendering the two notions amenable of interactions not allowed in the classical world.

Useful tools for inquire into possible interplays of vagueness and uncertainty in the quantum realm are provided by *quantum computational logics*. These logics, investigated by Maria Luisa Dalla Chiara, Roberto Giuntini and other authors, including the present writers [1, 2, 3, 4, 5, 6, 8], differ from the well known Birkhoff-von Neumann approach to quantum logic, where propositions ascribing properties are represented by projection operators (or, equivalently, by closed subspaces of a Hilbert space). In quantum computational logics meanings of sentences are no longer represented as projector operators on a Hilbert space, but by means of *quantum information quantities*: qubits, qutrits, density operators. In this framework fuzzy-like structures appear at different levels, and with different status. The aim of the present talk is to discuss this further bridge between many-valued and quantum logics.

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Rossella Marrano

A qualitative perspective on vagueness and degrees of truth

One of the primary goal of infinite-valued logics (see e.g. [1]) is to model reasoning under vagueness. In such an approach, sentences containing borderline predications, which are taken to be neither true nor false, are assigned intermediate, non-classical truth-values. Moreover, in order to capture the soritical aspect typical of the phenomenon of vagueness, a continuum of values is considered. To this aim the real unit interval $[0, 1]$ is taken as the set of truth-values and those values are then interpreted as degrees of truth (see e.g. [3]).

The just outlined degree-theoretic approach toward vagueness has been the object of longstanding criticisms (see e.g. [2]). In particular, it has been argued that the idea of evaluating each sentence to a unique real number is inadequate to model vagueness, as it involves both arbitrariness of the choice (how can we justify the choice of the truth value 0.24 over 0.23?) and implausibility of the interpretation (what does it mean for a sentence to be $1/\pi$ true?). This argument is known as *artificial precision objection* because it claims that the assignment of an exact number as truth-value imposes a precision which is unacceptable for sentences involving vague predicates.

The main contribution of this paper is to argue that this and related difficulties can be overcome by adopting a qualitative perspective on modelling degrees of truth. The key step to do this consists in shifting the focus from the point-wise evaluation of sentences, which typically features in degree-theoretic approaches to vagueness, to the binary comparison of their truth-values.

In order to have a better grasp on this, let $\mathcal{SL}$ be a propositional language and consider Łukasiewicz real-valued logic with its standard truth-value semantics. This paper puts forward an alternative semantics based on a binary relation on the set of sentences $\preceq\, \subseteq \mathcal{SL} \times \mathcal{SL}$ interpreted as “no more true than”. Building on ongoing work, we can lay down sufficient conditions for this relation to represent a valuation in Łukasiewicz logic. More precisely, we put forward an axiomatisation of $\preceq$ so that there exists $v\colon \mathcal{SL} \to [0, 1]$ such that for all $\theta, \phi \in \mathcal{SL}$

$$\theta \preceq \phi \Rightarrow v(\theta) \leq v(\phi)$$

where v is a Łukasiewicz valuation and $\leq$ the natural order on the real numbers.

This result sets the conditions under which a quantitative evaluation arises from qualitative comparisons. In addition, we argue in favour of the plausibility of the axioms given the interpretation of $\preceq$. In virtue of the qualitative foundation the commitment to a unique numerical assignment for sentences is shown not to be necessary, so that the objection of the artificial precision loses much of its force.

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Francesco Paoli

Vagueness, Equivocation, and Truth Degrees

In this talk, we defend a degree-theoretical approach to vagueness which is very close in spirit to Nick Smith's fuzzy plurivaluationism (see e.g. N.J. Smith, Vagueness and Degrees of Truth, Oxford University Press, 2008). We argue that classical logic is inappropriate to cope with vagueness, but this is not because it is deductively too strong: rather, it is expressively inadequate, in that it collapses logical constants that turn out to be distinct in logics that admit truth degrees. In particular, we reconstruct the sorites in Rational Pawelka Logic (RPL) and we contend that it is a fallacy of equivocation. Its conditional premisses, in fact, are ambiguous between a "closeness" reading (on which some of these premisses are not fully true and the argument is unsound) and a "tolerance" reading (on which modus ponens fails and the argument is invalid). To formalise the latter reading, we use a family of conditionals A $\rightarrow_r$ B, indexed by the rational numbers in (0,1]. Informally, A $\rightarrow_r$ B is fully true iff it is true at least to degree $1 - r$ that A implies B. In the spirit of fuzzy plurivaluationism, we argue that there is no fact of the matter as to which exact value r has in the previous schema: there can be different values in different acceptable interpretations. Finally, we briefly discuss a purely structural version of the sorites due to Elia Zardini.

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Dave Ripley

Two kinds of higher-order vagueness

I explore the relations between two kinds of higher-order vagueness (hov). For my purposes, higher-order vagueness is vagueness in the theoretical terms we use to understand vague predicates.

The first kind is what I will call 'borderline hov': it is the familiar vagueness of the predicate 'borderline P', when P is itself a vague predicate.

The second kind is what I will call 'similarity hov': this is less discussed, but I think no less important. When we think about the principle of tolerance that a vague predicate P seems to obey, we see a notion of similarity. Tolerance says that if two things are sufficiently similar (in certain respects), then if one of them is P, so too is the other. Similarity hov is what we see when we note that 'sufficiently similar (in certain respects)' is itself vague.

I will argue that both kinds of higher-order vagueness are real, and draw some lessons about the relationship between borderline cases and tolerance.

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Nick Smith

Truth, Probability of Truth and Truth-Functionality

A common criticism of degree-based approaches to vagueness — especially from those who favour supervaluationist approaches — centres on the truth-functionality inherent in many degree-based accounts. Elsewhere I have presented a defence of degree-based approaches against these objections. Here I go on the offensive and argue that any correct model of truth must render certain connectives truth-functional (in certain circumstances). Hence supervaluationism, in particular, cannot provide a good model of truth. Supervaluationism may be a model of probability of truth, or even of assertibility — but it cannot be an adequate model of truth: truth cannot be supertruth.

Satoru Suzuki

A Measurement-Theoretic Approach to Observational-Predicate Logic

Motivation

Vagueness is a ubiquitous feature that we know from many expressions in natural languages. It can invite a serious problem: the **Sorites Paradox**. We **specify** the sort of Sorites Paradox we tackle in this talk. Graff (2001) defines an observational predicate as follows:

Definition 1 (Observational Predicate) *A predicate is **observational** iff its applicability to an object (given a fixed context of evaluation) depends only on the way that object appears. For example, 'looks-red', 'sounds-loud', and 'tastes-sweet'.*

Observational predicates can generate a version of the Sorites Paradox called the **Phenomenal Sorites Paradox**. By modifying Graff (2001), we can show the defining features of the Phenomenal Sorites Paradox as follows: (1) the occurrence of some kind of **tolerance principle on perceptual indiscriminability**, (2) the occurrence of some expressions for perceptual indiscriminability—'looks-the-same-as' or 'smells-the-same-as', etc.—in the antecedent of the tolerance principle, (3) the occurrence of **observational predicates** as the other constituents of the argument, and (4) the occurrence of some kind of **premises on indiscriminability**. According to Raffman (2000), we can classify perceptual indiscriminability as follows: (1) s**-indiscriminability**: perceptual indiscriminability in the **statistical** sense, and (2) d**-indiscriminability**: perceptual indiscriminability in the **non-statistical** (**dispositional**) sense. The standard model of economics is based on **global rationality** that requires an **optimising behavior**. But according to Simon (1982), cognitive and information-processing constrains on the capabilities of agents, together with the complexity of their environment, render an **optimising behavior** an **unattainable ideal**. He dismissed the idea that agents should exhibit global rationality and suggested that they in fact exhibit **bounded rationality** that allows a **satisficing behavior**. If an agent has only a **limited** ability of discrimination, he may be considered to be only **boundedly rational**. We shall discuss s**-Indiscriminability**. If an agent is **boundedly-rational**, one possible explanation for this paradox is that the **nontransitivity** of s-indiscriminability results from the fact that he **cannot generally** discriminate very close quantities. The psychophysicist Fechner (1860) explained this inability by the concept of a **threshold of discrimination**, that is, **just noticeable difference** (**JND**). Given a measure function f that an experimenter could assign to a boundedly-rational agent and an object a, its JND δ is the **lowest intensity increment** such that $f(a)+\delta$ is recognised to be higher than $f(a)$ by the agent. We can consider the notion of a JND from a **statistical** point of view. The JND is usually the difference that a **boundedly-rational agent** makes on 50% of trials. If a different proportion from **50**% is used, then this should be included in the description—for example, "**75% JND**". We define the Tolerance Principle on s-Indiscriminability as follows:

Definition 2 (Tolerance Principle on s-Indiscriminability) *For any object x, y, if an **examiner** B makes a **statistical** judgment that x **looks the same as** y to an **examinee** A in the respect of the property expressed by a **observational predicate** F, then if $F(x)$, then $F(y)$.*

The Phenomenal Sorites Paradox on s-Indiscriminability is as follows:

Example 1 (Phenomenal Sorites Paradox on s-Indiscriminability) • *Patch 1 looks red to an examinee A.*

• *(**Tolerance Principle**): For any patch x, y, if an examiner B makes a statistical judgment that x looks the same as y to A, then if x looks red to A, then y looks red to A.*

• *(**Premise on Indiscriminability** 1): B makes a statistical judgment that Patch 1 looks the same as Patch 2 to A.*

• *(**Premise on Indiscriminability** 2): B makes a statistical judgment that Patch 2 looks the same as Patch 3 to A.*

⋮

• *(**Premise on Indiscriminability** 99): B makes a statistical judgment that Patch 99 looks the same as Patch 100 to A.*

Patch 100 looks red to A.

The Tolerance Principle on s-Indiscriminability can be **false** because the objects that are the **same** may often be recognized **different** by an **examinee** A and the objects that are **different** may often be recognized the **same** by A. The Premises on s-Indiscriminability are all true because of his **limited ability**.

The argument of this example is **valid**. On the other hand, we shall discuss d-**Indiscriminability**. We define the Tolerance Principle on d-Indiscriminability as follows:

Definition 3 (Tolerance Principle on d-Indiscriminability) *For any object x, y and any **context** z, if an agent A **would make a judgment** that x **looked the same** as y to A in the respect of the property expressed by an **observational predicate** F in z if he were to compare x with y in z, then if $F(x)$, then $F(y)$.*

The Phenomenal Sorites Paradox on d-Indiscriminability becomes as follows:

Example 2 (Phenomenal Sorites Paradox on d-Indiscriminability) • *Patch 1 looks red to an agent A in a context C_1.*
• *(**Tolerance Principle**): For any patch x, y and any context z, if A would make a judgment that x looked the same as y to A in a context z if A were to compare x with y in z, then if x looks red to A, then y looks red to A.*
• *(**Premise on Indiscriminability** 1): A would make a judgment that Patch 1 looked the same as Patch 2 to A in a context C_1 if A were to compare Patch 1 with Patch 2 in C_1.*
• *(**Premise on Indiscriminability** 2): A would make a judgment that Patch 2 looked the same as Patch 3 to A in C_2 if A were to compare Patch 2 with Patch 3 in C_2.*
⋮
• *(**Premise on Indiscriminability** 99): A would make a judgment that Patch 99 looked the same as Patch 100 to A in C_{99} if A were to compare Patch 99 with Patch 100 in C_{99}.*

Patch 100 looks red to A in C_{99}.

In this example, we agree with Graff (2001) and Raffman (2000) in thinking that there can occur different d-indiscriminability relations **relative to contexts** such as $C_1, C_2, \ldots, C_{99}$ since if an agent is boundedly rational, he cannot necessarily **attend to** all patches **simultaneously** because of his limited ability of discrimination even if he can have them all **in view simultaneously**. Because it may not be that any of the d-indiscriminability relations are the same, it is **meaningless** to argue whether a **fixed** d-indiscriminability relation is **transitive** or not. If $C_1 = C_2 = \ldots = C_{99}$ held, the fixed d-indiscriminability relation **would be transitive**. Because, in a **single observation**, the objects that are judged **different** by an agent A are **different** for A and the objects that are judged the **same** by A are the **same** for A, the Tolerance Principle on d-indiscriminability is **true**. The Premises on d-Indiscriminability are true because of his **limited ability** of discrimination. The argument of this example is **invalid** because there can occur **different** d-indiscriminability relations relative to contexts such as $C_1, C_2, \ldots, C_{99}$. The characteristics of the Phenomenal Sorites Paradoxes can be schematized as follows:

	s-Indiscriminability	d-Indiscriminability
Indiscriminability Relations	**Fixed** and **Nontransitive**	**Possibly Different**
Tolerance Principle	**False**	**True**
Premises on Indiscriminability	True	True
Argument	**Valid**	**Invalid**

Hyde (2005) classified responses to the Sorites Paradox in the following four types: (1) **denying that logic applies to soritical expressions**, (2) **denying some premises, including a Tolerance Principle**, (3) **denying the validity of the argument**, and (4) **accepting the paradox as sound**. From the consideration above, we conclude that we make a **response (3)** to the Phenomenal Sorites Paradox on d-Indiscriminability and that we make a **response (2)** to the Phenomenal Sorites Paradox on s-Indiscriminability. The **aim of this talk** is to propose a new version of logic for **observational** predicates—Observational-Predicate Logic (OPL)—that can avoid the **Phenomenal Sorites Paradox on s-Indiscriminability**. To accomplish this aim, we provide the language of OPL with a **statistical** model in terms of **measurement theory**. Numerous studies (for example, Raffman (2000), Graff(2001), and Keefe (2011)) have been made on the Phenomenal Sorites Paradox on d-Indiscriminability. But only few attempts have so far been made at the Phenomenal Sorites Paradox on s-Indiscriminability. Indeed Hardin (1988) discussed the Phenomenal Sorites Paradox on s-Indiscriminability in terms of JNDs, but he dealt with it neither from a logical point of view nor from a measurement-theoretic one.

Statistical Consistency: Strong Statistical Transitivity (SST)

When $\mathcal{I}$ is a nonempty set of individuals, we define a **forced-choice-pair-comparison probability function** as follows:

Definition 4 (Forced-Choice-Pair-Comparison Probability Function Pr) *$Pr : \mathcal{I} \times \mathcal{I} \to [0, 1]$ is called a **forced-choice-pair comparison probability function** if it satisfies the following condition: For any $x, y \in \mathcal{I}$ such that $x \neq y$, $Pr(x, y) + Pr(y, x) = 1$.*

Remark 1 (Relative Frequency) *$Pr(a, b)$ is interpreted as the **relative frequency** with which an agent will choose a rather than b when **forced** to make a choice from $\{a, b\}$.*

The following is one of the most typical conditions for **statistical consistency**.

Definition 5 (Strong Statistical Transitivity (SST)) *Pr is said to satisfy the **Strong Statistical Transitivity (SST)** if for any $x, y, z \in \mathcal{I}$, if $Pr(x, y) \geq \frac{1}{2}$ and $Pr(y, z) \geq \frac{1}{2}$, then $Pr(x, z) \geq \max\{Pr(x, y), Pr(y, z)\}$.*

The following is an example of SST.

Example 3 (Phenomenal Sorites Paradox on s-Indiscriminability and SST) *Suppose that an examiner observes the relative frequency with which an examinee responds that Patch i $(1 \leq i \leq 100)$ looks different from Patch j $(1 \leq j \leq 100)$. For example, when the relative frequency with which the examinee responds that Patch 50 looks different from Patch 52 is $\frac{3}{4}$ and that with which he responds that Patch 52 looks different from Patch 54 is $\frac{2}{3}$, it is plausible that the relative frequency with which he responds that Patch 50 looks different from Patch 54 should be at least $\frac{3}{4}$. Then these relative frequencies should satisfy **SST**.*

Measurement-Theoretic Analysis of JNDs and Semiorders

Luce (1956) introduced the concept of a **semiorder** that can provide a **qualitative** counterpart of a JND. Scott and Suppes (1958) defined a semiorder as follows:

Definition 6 (Semiorder) *$\succ$ on $\mathcal{I}$ is called a semiorder if, for any $w, x, y, z \in \mathcal{I}$, the following conditions are satisfied: (1) **Irreflexivity**: $x \not\succ x$, (2) **Intervality**: If $w \succ x$ and $y \succ z$, then $w \succ z$ or $y \succ x$, and (3) **Semitransitivity**: If $w \succ x$ and $x \succ y$, then $w \succ z$ or $z \succ y$.*

There are two main problems with **measurement theory**: (1) the **representation problem**: justifying the assignment of numbers to objects, and (2) the **uniqueness problem**: specifying the transformation up to which this assignment is unique. A solution to the former can be furnished by a **representation theorem**, which establishes that the specified conditions on a qualitative relational system are (necessary and) sufficient for the assignment of numbers to objects that represents (or preserves) all the relations in the system. Scott and Suppes (1958) proved a representation theorem for semiorders when $\mathcal{I}$ is **finite**. The Scott-Suppes theorem was first extended by to **countable** sets by Manders (1981). Because $\mathcal{I}$ of the model $\mathfrak{M}$ of the $\mathcal{L}_{\mathsf{OPL}}$ may be countable, the Manders theorem must be considered. A condition (**$\sim^*$-Connectedness**) is necessary for $\succ$ to have a positive threshold even when $\mathcal{I}$ is **countable**. $\sim$ is defined as follows:

Definition 7 ($\sim$) *For any $x, y \in \mathcal{I}$, $x \sim y := x \not\succ y$ and $y \not\succ x$.*

$\sim^*$ is defined by $\sim$ and $\succ$ as follows:

Definition 8 ($\sim^*$) *For any $x, y \in \mathcal{I}$, $x \sim^* y := (x \sim y)$ or ($x \succ y$ and for any $z \in \mathcal{I}$, not ($x \succ z$ and $z \succ y$)) or ($y \succ x$ and for any $z \in \mathcal{I}$, not ($y \succ z$ and $z \succ x$)).*

A $\sim^*$-chain is defined by $\sim^*$ as follows:

Definition 9 ($\sim^*$-Chain) *Let $a_0, \ldots, a_n \in \mathcal{I}$ be such that for any $k < n, a_k \sim^* a_{k+1}$. Then we call $(a_0, \ldots, a_n)$ a $\sim^*$-chain between a_0 and a_n.*

$\sim^*$-Connectedness is defined by a $\sim^*$-chain as follows:

Definition 10 ($\sim^*$-Connectedness) *$\sim^*$ on $\mathcal{I}$ is connected if for any $x_0, x_n \in \mathcal{I}$, there is a $\sim^*$-chain between x_0 and x_n.*

The Manders theorem can be stated by means of $\sim^*$-Connectedness as follows:

Theorem 1 (Representation for Semiorders, Manders (1981)) *Suppose that $\succ$ is a binary relation on a countable set $\mathcal{I}$ and that $\sim^*$ is defined by Definition 8 and that δ is a positive number (**JND**). Then $\succ$ is a semiorder and $\sim^*$ is connected iff there is a function $f : \mathcal{I} \to \mathbb{R}$ such that for any $x, y \in \mathcal{I}$, $x \succ y$ iff $f(x) > f(y) + \delta$.*

We define a binary relation Pr^λ on $\mathcal{I}$ as follows:

Definition 11 (Binary Relation Pr^λ on $\mathcal{I}$) *Pr^λ is a binary relation on $\mathcal{I}$ such that for any $x, y \in \mathcal{I}$, $xPr^\lambda y$ if $Pr(x, y) > \lambda$.*

Now we consider the notion of a **JND** from a **statistical** point of view. As we have mentioned before, the JND is usually the difference that a boundedly-rational agent makes on $\frac{1}{2}$ **or more** of trials. Here we consider the JND in terms of the family $\{Pr^\lambda : \lambda \in [\frac{1}{2}, 1)\}$. We define some concepts as follows:

Definition 12 (Compatibility, Homogeneousness and Discriminatedness) *A **semiorder** $\succ$ and a **weak order** $\succeq$ are said to be **compatible** if the following conditions hold: for any $z, y, z \in \mathcal{I}$, if $x \succ y$, then $x \succeq y$, and if $x \succeq y \succeq z$ and $x \sim z$, then $x \sim y$ and $y \sim z$. The family of semiorders is called **homogeneous** if the same weak order is compatible with each member of the family. Pr is called **discriminated** if for any $x, y \in \mathcal{I}$, if $Pr(x, y) = \frac{1}{2}$, then for any $z \in \mathcal{I}$, $Pr(x, z) = Pr(y, z)$.*

Roberts (1971) proved the following theorem concerning SST and homogeneous family of semiorders:

Theorem 2 (SST and Homogeneous Family of Semiorders, Roberts (1971)) *Suppose that Pr is a **discriminated** forced-choice-pair-comparison probability function. Then Pr satisfies **SST** iff $\{Pr^\lambda : \lambda \in [\frac{1}{2}, 1)\}$ is a **homogeneous** family of **semiorders**.*

We have the following corollary of Theorem 1 and Theorem 2.

Corollary 1 (Representation for Pr^λ) *Suppose that Pr is a **discriminated** pair comparison probability function. Then Pr satisfies **SST** and the relation obtained by Definition 8 from each member of $\{Pr^\lambda : \lambda \in [\frac{1}{2}, 1)\}$ is **connected** iff for any $Pr^\lambda \in \{Pr^\lambda : \lambda \in [\frac{1}{2}, 1)\}$that is a **homogeneous** family of **semiorders**, there is a function $f : \mathcal{I} \to \mathbb{R}$ such that for any $x, y \in \mathcal{I}$, $xPr^\lambda y$ iff $f(x) > f(y) + \delta$.*

Observational-Predicate Logic (OPL)

We define the language $\mathcal{L}_{\mathsf{OPL}}$ of OPL as follows:

Definition 13 (Language of OPL) *Let $\mathcal{V}$ denote a set of individual variables, $\mathcal{C}$ a set of individual constants, $\mathcal{P}$ a set of one-place **observational predicate symbols**, and $\gtrless_P$ a s-**Discriminability relation symbol** relative to $P \in \mathcal{P}$. The language $\mathcal{L}_{\mathsf{OPL}}$ of OPL is given by the following BNF grammar:*

$t ::= x \mid a,$

$\varphi ::= P(t) \mid t_i = t_j \mid t_i \gtrless_P t_j \mid \top \mid \neg\varphi \mid \varphi \wedge \psi \mid \forall x\varphi,$

*where $x \in \mathcal{V}$, $a \in \mathcal{C}$, $P \in \mathcal{P}$. $\bot, \vee, \to, \leftrightarrow$ and $\exists$ are introduced by the standard definitions. $t_i \gtrless_P t_j$ means that an **examiner** B makes a **statistical** judgment that an **examinee** A can **discriminate** t_i in P-ness from t_j. A s-**Indiscriminability relation symbol** $t_i \approx_P t_j$ relative to P is defined as $\neg(t_i \gtrless_P t_j)$. The set of all well-formed formulae of $\mathcal{L}_{\mathsf{OPL}}$ is denoted by $\Phi_{\mathcal{L}_{\mathsf{OPL}}}$.*

On the basis of SST, we define a **statistical** model $\mathfrak{M}$ of $\mathcal{L}_{\mathsf{OPL}}$ as follows:

Definition 14 (Statistical Model $\mathfrak{M}$ of $\mathcal{L}_{\mathsf{OPL}}$) *$\mathfrak{M}$ is a tuple $(\mathcal{I}, a^{\mathfrak{M}}, b^{\mathfrak{M}}, \ldots, F^{\mathfrak{M}}, G^{\mathfrak{M}}, \ldots, Pr^{\lambda_1}_{F^{\mathfrak{M}}}, Pr^{\lambda_2}_{G^{\mathfrak{M}}}, \ldots)$, where: (1) $\mathcal{I}$ is a nonempty set of individuals, called the universe of $\mathfrak{M}$, (2) $a^{\mathfrak{M}}, b^{\mathfrak{M}}, \ldots \in \mathcal{I}$, (3) $F^{\mathfrak{M}}, G^{\mathfrak{M}}, \ldots \subseteq \mathcal{I}$, (4) $Pr_{F^{\mathfrak{M}}} : \mathcal{I} \times \mathcal{I} \to [0, 1]$ is a **discriminated forced-choice-pair-comparison probability function** relative to $F^{\mathfrak{M}}$ that represents the **relative frequency**, which an **examiner** B observes, with which an **examinee** A responds relative to $F^{\mathfrak{M}}$ and satisfies the **SST**,…, (5) $Pr^{\lambda_1}_{F^{\mathfrak{M}}}$ is a binary relation on $\mathcal{I}$ such that for any $x, y \in \mathcal{I}$, $xPr^{\lambda_1}_{F^{\mathfrak{M}}}y$ if $Pr_{F^{\mathfrak{M}}}(x, y) > \lambda_1$, where $\lambda_1 \in [\frac{1}{2}, 1)$,…, and (6) The relation obtained by Definition 8 from $Pr^{\lambda_1}_{F^{\mathfrak{M}}}$ is **connected**,….*

We provide $\mathcal{L}_{\mathsf{OPL}}$ with the following satisfaction definition relative to $\mathfrak{M}$:

Definition 15 (Satisfaction) *When an assignment function $s : \mathcal{V} \to \mathcal{I}$ is given, what it means for $\mathfrak{M}$ to satisfy $\varphi \in \Phi_{\mathcal{L}_{\mathsf{OPL}}}$ with s, in symbols $\mathfrak{M} \models_{\mathcal{L}_{\mathsf{OPL}}} \varphi[s]$ is inductively defined as follows: (1) The satisfaction clauses of $P, =, \top, \neg, \wedge$ and $\forall$ are standard ones, (2) $\mathfrak{M} \models_{\mathsf{OPL}} t_i \gtrless_P t_j[s]$ iff $\tilde{s}(t_1)Pr^{\lambda}_{P^{\mathfrak{M}}}\tilde{s}(t_2)$ or $\tilde{s}(t_2)Pr^{\lambda}_{P^{\mathfrak{M}}}\tilde{s}(t_1)$, $\lambda \in [\frac{1}{2}, 1)$.*

The next corollary follows from Corollary 1 and Definition 15.

Corollary 2 (Positive Threshold) *(1) There is a function* $f : \mathcal{I} \to \mathbb{R}$ *such that* $\mathfrak{M} \models_{\mathsf{OPL}} t_1 \gtrless_P t_2[s]$ *iff* $\tilde{s}(t_1) Pr^{\lambda}_{P^{\mathfrak{M}}} \tilde{s}(t_2)$ *or* $\tilde{s}(t_2) Pr^{\lambda}_{P^{\mathfrak{M}}} \tilde{s}(t_1)$ *iff* $f(\tilde{s}(t_1)) > f(\tilde{s}(t_2)) + \delta$ *or* $f(\tilde{s}(t_2)) > f(\tilde{s}(t_1)) + \delta$, *(2) There is a function* $f : \mathcal{I} \to \mathbb{R}$ *such that* $\mathfrak{M} \models_{\mathsf{OPL}} t_1 \approx_P t_2[s]$ *iff not* $\tilde{s}(t_1) Pr^{\lambda}_{P^{\mathfrak{M}}} \tilde{s}(t_2)$ *and not* $\tilde{s}(t_2) Pr^{\lambda}_{P^{\mathfrak{M}}} \tilde{s}(t_1)$ *iff* $f(\tilde{s}(t_1)) - \delta \leq f(\tilde{s}(t_2)) \leq f(\tilde{s}(t_1)) + \delta$.

Remark 2 (Positive Threshold and Bounded Rationality) *Because this corollary implies that an examinee has a* ***positive threshold*** *(only limited ability) of discrimination, he may be considered to be only* ***boundedly rational****.*

We now return to the Phenomenal Sorites Paradox on s-Indiscriminability. Assume that $\mathfrak{U} := (\mathcal{I}, a_1^{\mathfrak{U}}, \ldots, a_{100}^{\mathfrak{U}}, R^{\mathfrak{U}}, Pr^{\lambda}_{R^{\mathfrak{U}}})$ is given, where: (1) $\mathcal{I} := \{a_1, \ldots, a_{100}\}$, (2) a_i denotes the i-th colour patch, for any $i (1 \leq i \leq 100)$ grading from red to yellow, (3) R denotes looking red to an examinee A, (4) $Pr_{R^{\mathfrak{U}}}$ is a discriminated forced-choice-pair-comparison probability function relative to $R^{\mathfrak{U}}$ that represents the relative frequency, which an examiner B observes, with which an examinee A responds relative to $R^{\mathfrak{U}}$ and satisfies SST, (5) not $a_1^{\mathfrak{U}} Pr^{\lambda}_{R^{\mathfrak{U}}} a_2^{\mathfrak{U}}$ and not $a_2^{\mathfrak{U}} Pr^{\lambda}_{R^{\mathfrak{U}}} a_1^{\mathfrak{U}}$, ..., not $a_{99}^{\mathfrak{U}} Pr^{\lambda}_{R^{\mathfrak{U}}} a_{100}^{\mathfrak{U}}$ and not $a_{100}^{\mathfrak{U}} Pr^{\lambda}_{R^{\mathfrak{U}}} a_{99}^{\mathfrak{U}}$, (6) $R^{\mathfrak{U}}(a_{50}^{\mathfrak{U}})$ and not $R^{\mathfrak{U}}(a_{51}^{\mathfrak{U}})$, and (7) $a_{100}^{\mathfrak{U}} Pr^{\lambda}_{R^{\mathfrak{U}}} a_1^{\mathfrak{U}}$.
Then we have the following proposition:

Proposition 1 (Non-Tolerance on $\approx_P$) $\mathfrak{U} \not\models_{\mathsf{OPL}} \forall x \forall y (x \approx_R y \to (R(x) \to R(y)))$.

Remark 3 (Avoidance of Phenomenal Sorites Paradox on s-Indiscriminability) *This proposition reveals that we can* ***avoid*** *the Phenomenal Sorites Paradox on s-Indiscriminability by embodying a* ***response (2)*** *of* ***Motivation****.*

We define $\approx_P^*$ that is the syntactic counterpart of $\sim^*$ as follows:

Definition 16 ($\approx_P^*$) $t_1 \approx_P^* t_2 := t_1 \approx_P t_2 \vee (t_1 \gtrless_P t_2 \wedge \forall x \neg (t_1 \gtrless_P x \wedge x \gtrless_P t_2))$.

$\sim^*$-Connectedness is necessary, as we have seen, for a semiorder $\succ$ to have a positive threshold even when $\mathcal{I}$ is **countable**. OPL has the following metalogical property:

Theorem 3 (Non-Expressibility of $\sim^*$-Connectedness) $\sim^*$*-Connectedness is* ***not expressible*** *in terms of* $\approx_P^*$ *in* $\mathcal{L}_{\mathsf{OPL}}$.

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Zach Weber

Sorites in Paraconsistent Mathematics

Several solutions to the sorties paradox use non-classical logic. But what becomes of the sorites when we turn tables, and pose the problem using non-classical logics? What if the candidate logic is too weak to be able to formulate the paradox to begin with? If for example one counts heaps of sand using a paraconsistent arithmetic, does any paradox still obtain? There would seem to be something awry if a logic were not strong enough to express the very problems that logic was invoked to address. I will consider these questions by developing some basic paraconsistent arithmetic to demonstrate what a sorties paradox looks like when fully recast. What is wanted is a formalism strong enough at least enough to prove a sorites contradiction, but weak enough to render the sorites harmless. This opens up wider questions about the methodological implications for shifting a problem by shifting its logic.